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REPRINT

On the Sensitivity of von Neumann
and Morgenstern Abstract Stable
Sets: The Stable and the
Individual Stable Bargaining Set

by
J. Greenberg



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On the Sensitivity of von Neumann and Morgenstern Abstract Stable Sets: The Stable and the Individual Stable Bargaining Set

J. Greenberg¹

Abstract. The purpose of this paper is twofold: First, to study the properties of the notions of the “stable” and “individual stable” bargaining sets (SBS and ISBS). Second, to point out the sensitivity of the von Neumann and Morgenstern (vN&M) abstract stable set to the dominance relation that is being employed: Insisting that each member of the coalition be made better off yields the SBS, while requiring that at least one member of the coalition is better off and all others are not worse off yields the ISBS. Rather surprisingly, the SBS and the ISBS may have an empty intersection.

We fully characterize both the SBS and the ISBS in 3-person games with transferable utilities, and we also show that in ordinally convex games these two sets coincide with the core. As a by-product we thus derive a new proof that such games have a nonempty core. The paper concludes with an open question.

1 Introduction

The purpose of this paper is twofold: First, to study the properties of the notions of the “stable” and “individual stable” bargaining sets. Second, and perhaps more important, the paper points out the sensitivity of the von Neumann and Morgenstern (vN&M) abstract stable set to the dominance relation that is being employed. More specifically, we shall distinguish between the following two dominance relationships:

- (i) A coalition may object to a proposed payoff only if each of its members can be made better off.
- (ii) It suffices that at least one member is better off while all others are at least as well off for a coalition to object to a proposed payoff.

Condition (i) gives rise to the stable bargaining set (SBS), while condition (ii) yields the notion of the individual stable bargaining set (ISBS). And, rather surprisingly, it

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will be shown that these two dominance relations might yield two very different vN&M abstract stable sets. In particular, the SBS and the ISBS may have an empty intersection, even in games with transferable utilities. In fact, such is always the case in 3-person games that have an empty core.

We fully characterize both the SBS and the ISBS in 3-person superadditive games with transferable utilities. It turns out that in such games both sets contain only Pareto optimal payoffs.

We also show that in ordinarily convex games both the SBS and the ISBS coincide with the core. Since for the general n -person superadditive game both sets are nonempty (Greenberg 1990, Theorem 6.5.6), we get, as a by-product, a new proof that ordinarily convex games have a nonempty core. (See Vilkov 1977 and Greenberg 1985.)

The various notions of the bargaining set were originally introduced (for games with side payments²) by Aumann and Maschler (1964). A payoff belongs to (the³) bargaining set if and only if every "objection" to it can be "countered". Aumann and Maschler insisted that the objection be directed towards one other specific coalition. Mas-Colell (1989) modified the bargaining set in two ways. First, any coalition can make objections (see footnote 3), and second, an objection need not be directed to a particular coalition, but rather, can be countered by any other coalition. As noted by Dutta et al. (1989), while each of these bargaining sets "does test objections against counter objections, it does not similarly test the counterobjections or any further objections, and in this sense it is not consistent" (p. 94). To amend this deficiency, they require that not only "objections", but also "counterobjections", "counter-counterobjections", etc. be "justified" or "credible", yielding the notion of the "consistent bargaining set" (CB)⁴. They show, however, that the CB might be empty, even for 4-person games with transferable utilities. (See section 5).

It turns out that the set of Pareto optimal payoffs in the ISBS coincides with the CB. This is quite remarkable since the notions of the SBS and the ISBS were first suggested within the framework of the theory of social situations (Greenberg 1990), which is a new and integrative approach to the study of formal models in the social and behavioral sciences. More specifically, the theory of social situations has two main ingredients. First, it offers a unified way to represent cooperative and noncooperative social environments – by means of "situations". Second, it offers a unified criterion for the recommendations, namely, that the "standard of behavior" (for the given situation) be "stable"⁵. Shitovitz (Greenberg 1990, Theorem 4.5) observed that one⁶ of the stability concepts in this theory (specifically, the "optimistic stable stand-

² For extensions of these notion to games without side payments see, e.g., Asscher (1976, 1977), Billera (1970), Greenberg (1979), and Peleg (1963).

³ In almost all applications of the "classical bargaining set", the coalition that makes the objection, as well as the coalition to whom this objection is made, consist of a single individual.

⁴ See Remark 2.6.

⁵ In contrast, "classical game theory" offers three distinct representations of a social environment, namely, games in extensive form, normal (or strategic) form, and characteristic function (or coalitional) form. Moreover, to each type of game, game theory offers an abundance of solution concepts whose underlying motivations differ considerably.

⁶ In fact, the only one.

ard of behavior") can formally be associated with a vN&M abstract stable set⁷. Using this result, some of the better known game-theoretic solution concepts⁸, as well as interesting new solution concepts⁹, were derived from the unique vN&M abstract stable sets that correspond to different negotiation processes (Greenberg 1990).

The relationship between the ISBS and the bargaining set is remarkable also because, as Shubik (1984, p. 348) notes:

"We should stress, however, that the [bargaining set] concerns the stability of a single imputation, whereas the vN&M concept concerns the stability of a set of imputations. In a certain sense, then, the bargaining set (like the core and the kernel) is not a solution but a set of solutions – the collectivity of all possible outcomes using the particular solution concept. In contrast, each stable set in toto is a single solution, and the collectivity of all outcomes using this concept (the union of all stable sets) is generally not a stable set."

The paper concludes with the following open question. As noted above, Dutta et al.'s example of a 4-person transferable utility game demonstrates that, in general, the CB is empty. Hence, ISBS need not contain Pareto optimal payoffs. In contrast, the SBS in this example does contain Pareto optimal payoffs. Whether this is always the case in superadditive games with transferable utilities remains an open question. An affirmative answer to this question will be particularly pleasing, since if the "real world" is to provide a guideline, then the notion of the SBS seems to be more appropriate than the ISBS (and hence than the CB): Individuals often insist on some compensation if the "status quo" is to be changed.

Walter Bossert and Abhijit Sengupta pointed out to me that Dutta and Ray (1989) and Sengupta and Sengupta (1992) are related works, in the sense that they, too, explore the sensitivity of solution concepts to the strict and the weak dominance relations.

For ease of exposition the paper does not (explicitly) use the terminology or tools from the theory of social situations. Rather, it is cast completely within classical game theory, making use of the concept of vN&M abstract stable set. Following the associate editor's suggestion, the reader is referred to Greenberg (1990) for the motivation behind the abstract systems that define the SBS and the ISBS, as well as for definitions of well-known game-theoretic concepts.

⁷ This notion was introduced by vN&M in a few pages at the end of the second edition (in 1947) of their classical book. They offered it purely as a mathematical extension of "the vN&M solution"; they neither motivated it, nor suggested an application of it. In contrast, the theory of social situations stemmed from the basic question of "rational choice".

⁸ Such as: The core and the vN&M solution in cooperative games, coalition-proof Nash equilibrium in normal form games and the set of subgame perfect equilibria in extensive form games.

⁹ Such as: Refinements of subgame perfect equilibria, equilibria when either contingent threats or irrevocable commitments can be made by either an individual or a group of individuals.

2 The Stable and Individual Stable Bargaining Sets

This section provides the formal definitions of the stable and the individual stable bargaining sets, and studies some of their properties.

Let N be a finite nonempty set, and let R be the set of all real numbers. A coalition is a *nonempty* subset of N . For a coalition $S \subset N$, R^S denotes the S -dimensional Euclidean space. If $x \in R^N$ and S is a coalition, then x^S denotes the restriction of x on S . Let S be a coalition and let $x^S, y^S \in R^S$. Then, $x^S \geq y^S$ if $x^i \geq y^i$ for all $i \in S$; $x^S > y^S$ if $x^S \geq y^S$ but $x^S \neq y^S$; and $x^S \gg y^S$ if $x^i > y^i$ for all $i \in S$. Recall the following definition:

Definition 2.1: An n -person game in characteristic function form (henceforth a game) is a pair (N, v) where N is the nonempty finite set of players and v is the characteristic function which assigns to every coalition $S \subset N$, a nonempty and compact subset of R^S , denoted $v(S)$.

A game (N, v) is called a *Quasi transferable utility (QTU) game*, if it satisfies the following ("nonlevelness") property:

For all $S \subset N$ and $x, y \in v(S)$, if $x < y$ then there exists $z \in v(S)$ such that $x \ll z$. A game (N, v) is called a *transferable utilities (TU) game*, and is denoted by (N, μ) , if for all $S \subset N$ there exists a nonnegative scalar, $\mu(S)$, such that $v(S)$ is given by: $v(S) = \{x \in R^S \mid \sum_{i \in S} x^i \leq \mu(S)\}$.

In this paper we shall be concerned only with QTU games. For a QTU game (N, v) , let $v^*(S)$ denote the set of all S -Pareto optimal payoffs in $v(S)$, that is,

$$v^*(S) = \{x \in v(S) \mid \text{there is no } y \in v(S) \text{ such that } y^i > x^i \text{ for all } i \in S\}.$$

(The compactness of $v(S)$, see Definition 2.1, implies that $v^*(S)$ is compact and nonempty.)

Evidently, there are many negotiation processes that can be employed by players in a characteristic function form game, (N, v) . Thus, many abstract systems, describing these negotiation processes, can be associated with (N, v) . (See Greenberg 1990, Chapter 6, for negotiation processes that lead to the core and the vN&M solution). This paper is concerned with the procedure where each player updates his reservation price according to the last offer that was made to him¹⁰. More specifically, assume that a payoff x is offered. Coalition S can object to x if there is an S -Pareto optimal payoff $y^S \in v(S)$ which makes each member strictly better off, that is, $y^S \gg x^S$. Members of $N \setminus S$ continue to believe in $x^{N \setminus S}$. The new modified offer then becomes $y = (y^S, x^{N \setminus S})$. Now, another coalition, T , may object to y , again, on the basis that there is a T -Pareto optimal payoff $z^T \in v(T)$ such that each member of T is strictly better off under z^T than he is under y , that is, $z^T \gg y^T$. The resulting new modified offer is then $z = (z^T, y^{N \setminus T})$, and the bargaining process continues in the

¹⁰ This procedure is reminiscent of the housing market, where every seller evaluates his house according to the last offer he had, even though it is possible that one potential buyer made several offers to different sellers, so that it is impossible for all sellers to get their reservation price.

same manner. Observe that the bargaining procedure described here is such that modified offers need no longer be feasible, i.e., need not belong to $v(N)$.

The above procedure can be described by the following abstract system¹¹ (D, \angle) , where $D \equiv R^N_+$, and for $x, y \in D$,

$$x \angle y \Leftrightarrow \exists S \subset N, y^S \in v^*(S), \quad y^S \gg x^S, \quad \text{and} \quad y^{N \setminus S} = x^{N \setminus S}.$$

The definition of the dominance relation \angle is traditional: it is customary to require that a coalition will object to a proposed payoff only if *all* of its members are made better off, since changing the "status quo" is costly, and individuals have to be compensated for doing so. It is, nevertheless, interesting to study the consequences of modifying this requirement, and allow coalitions to object to a proposed payoff whenever at least one member is better off while all others are not worse off. Define, therefore, the second abstract system (D, \angle^*) as follows: $D \equiv R^N_+$, and for $x, y \in D$,

$$x \angle^* y \Leftrightarrow \exists S \subset N, y^S \in v^*(S), \quad y^S > x^S, \quad \text{and} \quad y^{N \setminus S} = x^{N \setminus S}.$$

Theorem 2.2: Both (D, \angle) and (D, \angle^*) admit unique vN&M abstract stable sets, A and A^* , respectively.

Proof: Greenberg 1990, Theorem 6.5.7.

The [individual] stable bargaining set is defined as the set of all feasible payoffs x , i.e., all payoffs $x \in v(N)$, which belong to the unique vN&M abstract stable set for (D, \angle) [(D, \angle^*)]. That is,

Definition 2.3: Let (N, v) be a QTU game. The *stable bargaining set (SBS)* of (N, v) is the set $SBS(N, v) \equiv A \cap v(N)$, and the *individual stable bargaining set (ISBS)* of (N, v) is the set $ISBS(N, v) \equiv A^* \cap v(N)$.

Theorem 2.4: Both the SBS and the ISBS are nonempty for all superadditive¹² games. Moreover, each of these two sets contains the core¹³ of the game.

Proof: Greenberg 1990, Theorems 6.5.4 and 6.5.6.

Rather surprisingly, the seemingly minor modification in the definition of the dominance relations might yield totally distinct solution concepts; the ISBS and the SBS may have an empty intersection, even for TU games! In particular, perhaps counter-intuitively, Example 3.10 shows that the SBS does not, in general, include the ISBS. However, Section 4 establishes that in ordinally convex games, the ISBS coincides with the SBS.

¹¹ For definitions and notation of well-known game-theoretic notions, as well as for motivation, see Greenberg 1990, Chapters 4 and 6.

¹² That is, (N, v) is such that for all $S, T \subset N, S \cap T = \emptyset$, we have that $x^S \in v(S)$ and $y^T \in v(T)$ imply $z \in v(S \cup T)$, where $z^i = x^i$ if $i \in S$, and $z^i = y^i$ if $i \in T$.

¹³ Note that the core of a QTU game is the same for both dominance relations \angle and \angle^* . Indeed, in QTU games, it is possible to make one player better off (all others as well off) if and only if it is possible to make all players better off.

Remark 2.5: The proof of Theorem 2.2 (which enabled the definitions of the SBS and the ISBS) makes use of von Neumann and Morgenstern's (1947, p. 597-600) result, asserting that if the dominance relation is strictly acyclic then there exists a unique abstract stable set. It is because of this that I require a coalition S to dominate by using S -Pareto optimal payoffs, while payoffs in SBS and ISBS were not required to belong to $v^*(N)$. An interesting question is whether A and A^* , respectively, remain (the unique?) vN&M abstract stable sets for the (possibly more appealing) systems $(D, \angle\angle)$ and $(D, \angle\angle^*)$, where $D \equiv R_+^N$, and for $x, y \in D$,

$$x \angle\angle y \Leftrightarrow \exists S \subset N, y^S \in v(S), y^S \gg x^S, \text{ and } y^{N \setminus S} = x^{N \setminus S}.$$

$$x \angle\angle^* y \Leftrightarrow \exists S \subset N, y^S \in v(S), y^S > x^S, \text{ and } y^{N \setminus S} = x^{N \setminus S}.$$

Remark 2.6: As was mentioned in the introduction, Dutta et al. (1989) offered the notion of the consistent bargaining set, $CB(N, v)$. These four scholars were motivated neither by the theory of social situations nor by vN&M abstract stable sets, but rather, they wanted to amend the deficiency in the definition of Mas-Colell's bargaining set¹⁴ by treating "objections" and "counterobjections" symmetrically. Ron Holtzman (private communication) observed that, as is easily verified, it turns out that their (recursive) definition is equivalent to: $CB(N, v) = ISBS \cap v^*(N)$. That is, $CB(N, v)$ is the set of Pareto optimal payoffs that belong to the individual stable bargaining set. Thus, as a by-product, we get a new characterization of the consistent bargaining set, based on vN&M abstract stable set. One of the advantages of this characterization is that it enables to extend this notion to games with an infinite number of players. (Recall that the original definition of the CB is recursive.) This is particularly appealing since it is market games with an atomless space of agents that initiated Mas-Colell's (1989) modified bargaining set. Of course, one has, then, to address the issues of existence and uniqueness of A and A^* for such games.

3 Three Players TU Games

Consider the 0-normalized TU game, (N, μ) , where $N = \{1, 2, 3\}$ and for all $i \in N$, $\mu(i) = 0$. (Recall that $\mu(S) \geq 0$ for all S ; see Definition 2.1.) Denote:

$$S_1 \equiv \{1, 2\}, \quad S_2 \equiv \{2, 3\}, \quad S_3 \equiv \{1, 3\},$$

and for all $x \in R_+^N$ and $S \subset N$,

¹⁴ Recall the following definitions: Let (N, v) be a cooperative game. A pair (S, y) is an *objection* to $x \in v(N)$ if $S \subset N$, $y \in R_+^N$, $y^S \in v^*(S)$, $y^S \gg x^S$, and $y^{N \setminus S} = x^{N \setminus S}$. Let (S, y) be an *objection* to x . (T, z) is a *counterobjection* to (S, y) if $T \subset N$, $z \in R_+^N$, $z^T \in v^*(T)$, $z^T \gg y^T$, and $z^{N \setminus T} = y^{N \setminus T}$. An objection (S, y) is *justified* if there does not exist any counterobjection to (S, y) . The (Mas-Colell) *modified bargaining set*, $MBS(N, v)$, consists of all payoffs in $v(N)$ for which there exists no justified objection.

$$x(S) \equiv \sum_{i \in S} x^i, \quad \text{and} \quad e(S, x) \equiv \mu(S) - x(S).$$

The function $e(\cdot, \cdot)$ is known as the *excess function*.

The two main results of this section fully characterize the SBS and the ISBS in 0-normalized TU three person games. Both sets contain only Pareto optimal payoffs and they both contain all of the core payoffs. But, for a non-core payoff to belong to the SBS it is necessary and sufficient that it be blocked by exactly two 2-players coalitions, while for a non-core payoff to belong to the ISBS it is necessary and sufficient that it be blocked by all three 2-players coalitions, and moreover, the excess of each of these coalitions is less than the sum of the excesses of the other two. It follows that if the core is empty, then the SBS and the ISBS are totally distinct sets. Formally,

Theorem 3.1: Let (N, μ) be a 0-normalized TU three person game. Then, $x \in \text{SBS}(N, \mu)$ if and only if, x is Pareto optimal, i.e., $x(N) = \mu(N)$, and either $x \in \text{Core}(N, \mu)$, or else, there exist $j, k \in \{1, 2, 3\}$ such that $e(S_j, x) = e(S_k, x) > 0$ and $e(S_{ij}, x) \leq 0$ for $\{i\} = \{1, 2, 3\} \setminus \{j, k\}$.

Theorem 3.2: Let (N, μ) be a 0-normalized TU three person game. Then, $x \in \text{ISBS}(N, \mu)$ if and only if, x is Pareto optimal, i.e., $x(N) = \mu(N)$, and either $x \in \text{Core}(N, \mu)$, or else, $e(S_j, x) > 0$ for $j = 1, 2, 3$, and moreover, $e(S_j, x) < e(S_k, x) + e(S_l, x)$, for any choice of j, k, l , $\{j, k, l\} = \{1, 2, 3\}$.

In order to establish Theorem 3.1 we first need the following three Lemmas.

Lemma 3.3: Let $y \in R_+^N$ be such that $y(N) \geq \mu(N)$, and there exists a choice of j, k, l , $\{j, k, l\} = \{1, 2, 3\}$, that satisfies: $e(S_j, y) = e(S_k, y)$, and $e(S_l, y) \leq 0$. Then, $y \in A$.

Proof: Otherwise, there exists $z \in A$ such that $y \prec z$. Since $e(S_l, y) \leq 0$, $e(N, y) \leq 0$, and the game is 0-normalized, it follows that $e(S_j, y) = e(S_k, y) > 0$. W.l.o.g., assume that z blocks y using S_j , i.e., $z(S_j) = \mu(S_j)$. Then, $e(S, z) \leq 0$ for all $S \neq S_k$, and $e(S_k, z) > 0$. Hence, there exists w such that $z \prec w$ using S_k , and $w(S) \geq \mu(S)$ for all $S \subset N$. Therefore, $w \in A$, which, together with the stability of A , contradicts $z \in A$.

Q.E.D.

Lemma 3.4: If $x \in A$ then there exists at least one coalition S , $|S| = 2$, such that $x(S) \geq \mu(S)$.

Proof: Otherwise, $e(S, x) > 0$ for all $|S| = 2$. Assume, w.l.o.g., that $e(S_1, x) \geq e(S_2, x) \geq e(S_3, x) > 0$. For α , $0 \leq \alpha \leq e(S_1, x)$, denote

$$y^\alpha \equiv (x_1 + \alpha, x_2 + [e(S_1, x) - \alpha], x_3).$$

Define the function $g, g: [0, e(S_1, x)] \rightarrow R$,

$$g(\alpha) \equiv e(S_2, y^\alpha) - e(S_3, y^\alpha) = e(S_2, x) - e(S_1, x) - e(S_3, x) + 2\alpha.$$

In particular,

$$g(0) = e(S_2, x) - e(S_1, x) - e(S_3, x) \leq -e(S_3, x) < 0,$$

and

$$g(e(S_1, x)) = e(S_2, x) - e(S_3, x) + e(S_1, x) \geq e(S_2, x) > 0.$$

Since g is continuous in α , there exists β , $0 < \beta < e(S_1, x)$, such that $g(\beta) = 0$. Define $y \equiv y^\beta$. Then, $x \angle y$ (using coalition $\{1, 2\}$). Since A is stable, $y \notin A$. By Lemma 3.3, [recall that $e(S_2, y) = e(S_3, y)$, and that for all α , $e(S_1, y^\alpha) = 0$], we have that $y(N) < \mu(N)$.

Define $\hat{y} \equiv (y_1, y_2, x_3 + [\mu(N) - y(N)])$. Then, $x \angle \hat{y}$ (using N). Since A is stable, $\hat{y} \notin A$. But, by Lemma 3.3, $\hat{y} \in A$ (since $\hat{y}(N) = \mu(N)$, $e(S_2, \hat{y}) = e(S_3, \hat{y})$, and $e(S_1, \hat{y}) = 0$). Contradiction. Q.E.D.

Lemma 3.5: If $x \in SBS$ then $x \in v^*(N)$, i.e., $x(N) = \mu(N)$.

Proof: Otherwise, $x \in SBS \setminus v^*(N)$. Assume, w.l.o.g., that $e(S_1, x) \geq e(S_2, x) \geq e(S_3, x)$. By Lemma 3.4, $e(S_3, x) \leq 0$. Hence, every coalition that blocks x contains player 2. Denote:

$$M \equiv \text{Max} \{e(S, x) \mid S \subset N\}, \quad B \equiv \{S \mid e(S, x) = M\}, \quad \text{and} \quad \delta \equiv [\mu(N) - x(N)]/3.$$

Then, $M > \delta > 0$. Distinguish among the following three cases:

1. $\{S_1, S_2\} \subset B$: Define y , where $y^i \equiv x^i + \delta$, $i = 1, 2, 3$. Clearly, $x \angle y$ (using N) and hence, $x \in A$ implies $y \notin A$. But, $e(S_1, x) = e(S_2, x) = M$ implies $e(S_1, y) = e(S_2, y)$, and therefore, (recall that $e(S_3, x) \leq 0$), by Lemma 3.3, $y \in A$. Contradiction.
2. $S_1 \notin B$: Then, $S \notin B$ for all $|S| = 2$. Thus, $B = \{N\}$. Recall that $e(S_3, x) \leq 0$. Thus, player 2 belongs to all the blocking coalitions. It follows, that there exists a payoff y , (with $y_2 > x_2 + \text{Max}\{e(S_1, x), 0\}$), such that $y \succ x$, $y(N) = \mu(N)$, and $y(S) \geq \mu(S)$ for all $S \subset N$. Thus, $y \in A$. But then, $x \angle y$ contradicts $x \in A$.
3. $S_1 \in B$ and $S_2 \notin B$: Then, there exists y that satisfies: $y_1 > x_1$, $y_2 > x_2$, $y_3 = x_3$, $y(1, 2) = \mu(1, 2)$, and $y(2, 3) \geq \mu(2, 3)$. Since $y(N) = x(N) + M \geq \mu(N)$, we have that $y \in A$, contradicting $x \angle y$ and $x \in A$. Q.E.D.

Proof of Theorem 3.1: Lemma 3.3 yields the “if” part of the theorem. To prove the “only if” part, it suffices, in view of Lemmas 3.4 and 3.5, to show that if x is Pareto optimal and satisfies: $e(S_j, x) > e(S_k, x) > 0$ and $e(S_i, x) \leq 0$ for $\{i\} = \{1, 2, 3\} \setminus \{j, k\}$, then $x \notin SBS(N, \mu)$. Indeed, since $S_j \cap S_k \neq \emptyset$, there exists $z \in v^*(S_j)$ such that $z^i > x^i$ for $i \in S_j$, $e(S_j, z) = 0 > e(S_k, z)$. Thus, $e(S, z) \leq 0$ for all S , implying that $z \in A$. Since $x \angle z$, we conclude that $x \notin A$, hence $x \notin SBS(N, \mu)$. Q.E.D.

We now turn to the proof of Theorem 3.2. In order to establish this theorem, we first need the following three Lemmas.

Lemma 3.6: If $x \in A^*$ and x can be blocked, then $x(S) < \mu(S)$ for all $|S| = 2$.

Proof: Otherwise, there exists $S \subset N$, $|S| = 2$ such that $e(S, x) \leq 0$. Assume, w.l.o.g., that $e(S_1, x) \geq e(S_2, x) \geq e(S_3, x)$. Then, $e(S_3, x) \leq 0$, implying that player 2 belongs to all the blocking coalitions. Define z , where $z_1 = x_1$, $z_3 = x_3$, and $z_2 = x_2 + \text{Max}\{e(S, x)\}$. Since x can be blocked, $z_2 > x_2$. Thus, $x \angle^* z$. But z cannot be blocked, hence $z \in A^*$, contradicting $x \angle^* z$ and $x \in A^*$. Q.E.D.

Lemma 3.7: If $x \in \text{ISBS}$ then $e(S_j, x) < e(S_k, x) + e(S_l, x)$, for any choice of j, k, l , $\{j, k, l\} = \{1, 2, 3\}$.

Proof: Otherwise, w.l.o.g., $e(S_1, x) \geq e(S_2, x) + e(S_3, x)$. Then, there exist y_1 and y_2 that satisfy: $y_1 \geq x_1 + e(S_1, x)$, $y_2 \geq x_2 + e(S_2, x)$, and $y_1 + y_2 = \mu(1, 2)$. Let z be equal to $y = (y_1, y_2, x_3)$ if $y(N) \geq \mu(N)$, and if $y(N) < \mu(N)$ let z be such that $z \gg y$, and $z(N) = \mu(N)$. Then, $z(S) \geq \mu(S)$ for all S , implying $z \in A^*$. But $x \angle^* z$, (using either N or $\{1, 2\}$), contradicting $x \in A^*$. Q.E.D.

Lemma 3.8: If $x \in \text{ISBS}$ then x is Pareto optimal, i.e., $x(N) = \mu(N)$. Alternatively, $\text{ISBS}(N, \mu)$ coincides with the consistent bargaining set, $\text{CB}(N, \mu)$.

Proof: Assume, in negation, that there exists $x \in \text{ISBS} \setminus v^*(N)$. Define,

$$\begin{aligned}\alpha_1 &\equiv \left(\frac{1}{2}\right)[e(S_1, x) + e(S_3, x) - e(S_2, x)] \\ \alpha_2 &\equiv \left(\frac{1}{2}\right)[e(S_1, x) + e(S_2, x) - e(S_3, x)] \\ \alpha_3 &\equiv \left(\frac{1}{2}\right)[e(S_2, x) + e(S_3, x) - e(S_1, x)] \\ y &\equiv (x_1 + \alpha_1, x_2 + \alpha_2, x_3 + \alpha_3).\end{aligned}$$

By Lemma 3.7, $y \gg x$. Now, if $y(N) \leq \mu(N)$, then there exists $z \geq y$ with $z(N) = \mu(N)$. It follows that $z(S) \geq \mu(S)$ for all S , implying that $z \in A^*$. But $x \angle^* z$, (using N), contradicts $x \in A^*$.

Thus, $y(N) > \mu(N)$. Since $x(N) < \mu(N)$, there exists z such that $z(N) = \mu(N)$, $z \gg x$ and $z \ll y$. Since $x \angle^* z$ (using N), $z \notin A^*$. That is, there exists $w \in A^*$ with $z \angle^* w$. Since $z(N) = \mu(N)$, w is supported by, w.l.o.g., $S_1 = \{1, 2\}$. That is, $w = (w_1, w_2, z_3)$, $w_1 \geq z_1$, $w_2 \geq z_2$, and $w(S_1) = \mu(S_1)$. Therefore, by Lemma 3.6, w cannot be blocked. In particular, $w(S_3) = w_1 + z_3 \geq \mu(1, 3)$ and $w(S_2) = w_2 + z_3 \geq \mu(2, 3)$. Hence, $w_1 + w_2 + 2z_3 = \mu(1, 2) + 2z_3 \geq \mu(1, 3) + \mu(2, 3)$, implying that $z_3 \geq \left(\frac{1}{2}\right)[\mu(1, 3) + \mu(2, 3) - \mu(1, 2)] = x_3 + \alpha_3 = y_3$. But $z \ll y$. Contradiction. Q.E.D.

Proof of Theorem 3.2: Let (N, μ) be a 0-normalized TU three person game. In view of Lemmas 3.6–3.8, it suffices to show that if $x \in v^*(N) \setminus \text{Core}(N, \mu)$ is such that $e(S, x) > 0$ for all $|S| = 2$, and moreover, $e(S_j, x) < e(S_k, x) + e(S_l, x)$, for any choice of j, k, l , $\{j, k, l\} = \{1, 2, 3\}$, then $x \in \text{ISBS}(N, \mu)$. Indeed, otherwise, there exists

$z \in A^*$ such that $x \prec^* z$. Since $x \in v^*(N)$, we have that $z(S_j) = \mu(S_j)$, for some $j \in \{1, 2, 3\}$. By Lemma 3.6, therefore, $e(S, z) \leq 0$ for all $|S| = 2$. But $e(S_j, x) < e(S_k, x) + e(S_i, x)$ implies that there exists k , such that $e(S_k, z) > 0$. Contradiction. Q.E.D.

Theorems 3.1 and 3.2 yield the following surprising result.

Corollary 3.9: Let (N, μ) be a 0-normalized 3-person TU game. Then, $A \cap A^* = \{x \in D \mid x \text{ is not blocked}\}$. It follows that $SBS \cap ISBS = Core(N, \mu)$. In particular, if the core is empty, then $SBS \cap ISBS = \emptyset$.

Proof: Since (N, μ) is a TU game, $\Delta(D) = \Delta^*(\Delta)^{15}$. By the (external) stability of A , $A \supset [D \setminus \Delta(D)]$. Similarly, by the (external) stability of A^* , $A^* \supset [D \setminus \Delta^*(D)]$. Hence, $A \cap A^* \supset [D \setminus \Delta(D)] = [D \setminus \Delta^*(D)]$.

To show that the reverse inclusion also holds, consider $x \in \Delta(D) [= \Delta^*(D)]$. By Lemma 3.6, if $x \in A^*$ then $x(S) < \mu(S)$ for every coalition S that consists of two players. By Lemma 3.4, therefore, $x \notin A$. Q.E.D.

I end this section by studying two examples that demonstrate some of the possible relationships among the three notions: The core, the SBS, and the ISBS. In both examples, I personally find the SBS to be more appealing than the ISBS¹⁶.

Example 3.10: The three person majority game: There are three players. Every coalition that has a (simple) majority, that is, every coalition consisting of two or more players, can distribute among its members 2 dollars. The utilities of the three players are linear with money. The TU game that describes this social environment is given by (N, μ) , where

$$N = \{1, 2, 3\}, \mu(S) = 2 \text{ if } |S| \geq 2, \text{ and for } i \in N, \mu(\{i\}) = 0.$$

As is well-known, $Core(N, \mu) = \emptyset$. Using Theorems 3.1 and 3.2, it is easy to see that

$$SBS(N, \mu) = \{(1, 1, 0), (1, 0, 1), (0, 1, 1)\},$$

and

$$ISBS(N, \mu) = \{x \mid x(N) = \mu(N) = 2, \text{ and } x_j < 1 \text{ for all } i \in N\}.$$

Thus, the core of this game is empty while the individual stable bargaining set, which coincides with the consistent bargaining set, is given by the shaded area in Figure 1. In contrast, the stable bargaining set consists of the three outcomes (the

¹⁵ For a set $B \subset D$, $\Delta(B)$ is the set of elements in D that are dominated by B , i.e., $\Delta(B) = \{x \in D \mid \text{there exists } y \in B, x \prec y\}$. Similarly, $\Delta^*(B) = \{x \in D \mid \text{there exists } y \in B, x \prec^* y\}$.

¹⁶ Recall that the ISBS is closely related to, and by Lemma 3.8 in 3-person games coincides with, the consistent bargaining set, $CB(N, \mu)$.

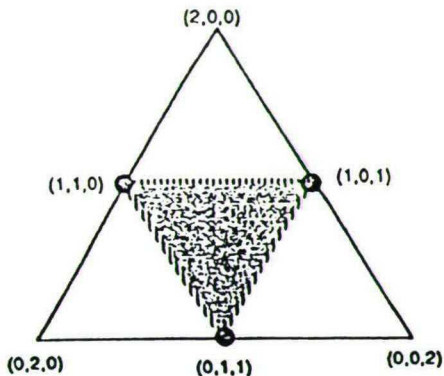


Fig. 1

three heavy dotted points in Figure 1) that stem from two players forming a coalition, and distributing the 2 dollars evenly between them.

Example 3.10 should not leave the impression that in general the SBS "is a smaller set" than the ISBS. Indeed, in the following example the SBS strictly includes the ISBS (implying, by Corollary 3.9 that the ISBS coincides with the core).

Example 3.11: Consider the game (N, μ) , where $N = \{1, 2, 3\}$, $\mu(1, 2) = \mu(1, 3) = \mu(N) = 100$, and $\mu(S) = \emptyset$ otherwise. Then,

$$\text{Core} = \text{ISBS} = \{(100, 0, 0)\} \quad \text{and} \quad \text{SBS} = \{(100 - 2\alpha, \alpha, \alpha) \mid 0 \leq \alpha \leq 50\}.$$

It is obvious that $\text{Core} = \{(100, 0, 0)\}$. The following observations verify the other assertions.

- (i) $\text{ISBS} = \{(100, 0, 0)\}$: Consider $x = (\alpha, \beta, \gamma) \in v(N)$ with $\alpha < 100$. Assume, w.l.o.g., that $\beta \geq \gamma$. Then, $(100 - \gamma) > \alpha$, implying that $y \equiv (100 - \gamma, \beta, \gamma)$ cannot be dominated. Hence y belongs to A^* . Since $x \prec^* y$, it follows that $x \notin \text{ISBS}$. Since $\text{Core} = \{(100, 0, 0)\}$, Theorem 2.4 yields that $\text{Core} = \text{ISBS} = \{(100, 0, 0)\}$.
- (ii) $\text{SBS} = \{(100 - 2\alpha, \alpha, \alpha) \mid 0 \leq \alpha \leq 50\}$: Observe, first, that $x = (\alpha, \beta, \gamma) \notin A$ whenever $\beta > \gamma$. Indeed, define, $\delta \equiv (\beta + \gamma)/2$, and $y \equiv (100 - \delta, \beta, \delta) \in A$. Then, y cannot be dominated. Hence y belongs to A . Since $x \prec y$, $x \notin A$. It follows that $x = (100 - 2\alpha, \alpha, \alpha) \in A$, since otherwise, there exists $z \in A$ such that $x \prec z$. But then $z = (z_1, z_2, z_3)$ with $z_2 > z_3$, and we just saw that such z cannot belong to A .

As in the previous example, I find the SBS for this game more appealing than the ISBS, despite the fact that the SBS is larger. It seems to me that players 2 and 3 are not totally "helpless" vis a vis player 1; they could, for example, collude.

4 Ordinally Convex Games¹⁷

There are, of course, games in which the SBS and the ISBS do intersect; in fact, there are games in which the two sets coincide. One class of such games is the set of "convex games", where

Definition 4.1: A game (N, v) is called

1. (ordinally) convex if it satisfies: For any $x \in R^N$ and any two coalitions S and T , if $x^S \in v(S)$ and $x^T \in v(T)$, then either $x^{S \cup T} \in v(S \cup T)$ or $x^{S \cap T} \in v(S \cap T)$.
2. comprehensive if for all $S \subset N$, $x \in v(S)$ implies $y \in v(S)$ for all $0 \leq y \leq x$. In particular, $0 \in v(S)$.

The main result of this section is

Theorem 4.2: Let (N, v) be a comprehensive QTU convex game. Then,

$$A = A^* = \{x \in R^N_+ \mid x \text{ cannot be blocked}\}.$$

In particular,

$$SBS(N, v) = ISBS(N, v) = Core(N, v).$$

In order to prove this theorem, we first need to establish the following Lemma.

Lemma 4.3: Let (N, v) be a comprehensive QTU convex game. Then,

$$F \equiv D \setminus \Delta(D) = \{x \in R^N \mid x \text{ cannot be blocked}\}$$

is a vN&M abstract stable set for (D, \angle) .

Proof: Clearly, $\Delta(F) \subset D \setminus F$. To conclude the proof we need to show that $D \setminus F \subset \Delta(F)$. The proof of this inclusion generalizes, but closely follows that of Proposition 3.3 in Dutta et al. Otherwise, there exists $x \in D \setminus F$ but $x \notin \Delta(F)$. Thus, there exists S that blocks x through some y , and $(y^S, x^{N \setminus S}) \notin F$. By the comprehensiveness of (N, v) , therefore, the set $Q_1 \neq \emptyset$, where

$$Q_1 \equiv \{(S, y) \mid y^S \in v^*(S), y^{N \setminus S} = x^{N \setminus S} \text{ and } y^S \succ x^S\}.$$

By considering minimal (in the set inclusion ordering) coalitions in Q_1 , it is easy to see that there exists $(\hat{S}, \hat{y}) \in Q_1$ such that no subset of \hat{S} can block \hat{y} , and \hat{S} is a maximal such coalition. Since $(\hat{y}^{\hat{S}}, x^{N \setminus \hat{S}}) \notin F$, the comprehensiveness of (N, v) implies that set $Q_2 \neq \emptyset$, where

¹⁷ This section makes extensive use of the results in Section 3.3 in Dutta et al. (1989).

$$Q_2 \equiv \{(T, z) \mid T \in v(T), z^{N \setminus (T \setminus \hat{S})} = \hat{y}^{N \setminus (T \setminus \hat{S})}, \text{ and } z^{T \setminus \hat{S}} \gg \hat{y}^{T \setminus \hat{S}}\}.$$

By the choice of \hat{S} , $(T, z) \in Q_2$ implies $T \setminus \hat{S} \neq \emptyset$. Let

$$(\hat{T}, \hat{z}) \in \text{Arg Max}_{\{(T, z) \in Q_2\}} \text{Min}_{\{i \in T \setminus \hat{S}\}} \hat{z}^i.$$

Since $(\hat{y}^{\hat{S}}, x^{N \setminus \hat{S}}) \notin F$, $\text{Min}_{\{i \in T \setminus \hat{S}\}} \hat{z}^i > 0$.

Then, $\hat{y}^{\hat{S}} = \hat{z}^{\hat{S}} \in v(\hat{S})$ and $\hat{z}^{\hat{T}} \in v(\hat{T})$. By the choice of \hat{S} , $\hat{z}^{\hat{S} \cap \hat{T}} = \hat{y}^{\hat{S} \cap \hat{T}} \notin \text{Int}.v(\hat{S} \cap \hat{T})$. By Lemma 3.3 in Dutta et al. (1989), it follows that $\hat{z}^{\hat{S} \cup \hat{T}} \in v(\hat{S} \cup \hat{T})$. Now, if there was a coalition $W \subset \hat{S} \cup \hat{T}$ that blocks \hat{z} , using $\omega \in v(W)$, then, by the choice of \hat{S} , $W \setminus \hat{S} \neq \emptyset$. Moreover, $\omega^{W \setminus \hat{S}} \gg \hat{z}^{W \setminus \hat{S}}$ implies that $\text{Min}_{\{i \in W \setminus \hat{S}\}} \omega^i > \text{Min}_{\{i \in T \setminus \hat{S}\}} \hat{z}^i$. Since $\omega^{\hat{S} \cap \hat{T}} \geq \hat{y}^{\hat{S} \cap \hat{T}}$ and (N, v) is comprehensive, the above strict inequality contradicts the choice of \hat{z} . Thus, \hat{z} is not blocked by any subset of $\hat{S} \cup \hat{T}$.

But this contradicts the maximality of \hat{S} . (Recall that $\hat{T} \setminus \hat{S} \neq \emptyset$, implying $|\hat{S} \cup \hat{T}| > |\hat{S}|$.) Q.E.D.

Proof of Theorem 4.2: By Lemma 4.3 we have that the unique vN&M abstract stable set for (D, \angle) is: $A = \{x \in R^N \mid x \text{ cannot be blocked}\}$. Using Theorem 2.2, in order to conclude the proof of the theorem, it suffices to show that A is a vN&M abstract stable set for (D, \angle^*) .

Indeed, since (N, v) is QTU, $A \cap \Delta(A) = \emptyset$ implies that $A \cap \Delta^*(A) = \emptyset$, that is, $\Delta^*(A) \subset D \setminus A$. To see that the reverse inclusion also holds, consider $x \in D \setminus A$. By Lemma 4.3, there exists $y \in A$ such that $x \angle y$, implying $x \angle^* y$. Hence, $\Delta^*(A) \supset D \setminus A$. Thus, $\Delta^*(A) = D \setminus A$, i.e., A is a vN&M abstract stable set for (D, \angle^*) . Q.E.D.

A by-product of Theorem 4.2 provides a new proof for the nonemptiness of the core of convex games (see Vilkov 1977, and Greenberg 1985).

Corollary 4.4: The core of a comprehensive QTU convex game is nonempty.

Proof: Let (N, v) be a comprehensive QTU convex game. Then, since $0 \in v(Q)$ for all $Q \subset N$, the convexity of (N, v) yields that for all $x^{\hat{S}} \in v(\hat{S})$, $(x^{\hat{S}}, 0^{N \setminus \hat{S}}) \in v(N)$. By Theorem 2.4, therefore, $\text{SBS}(N, v) \neq \emptyset$. Theorem 4.2 concludes the proof. Q.E.D.

It is noteworthy that Peleg (1986) proved that in convex games the core is also the unique vN&M solution.

5 An Open Question

By Theorem 2.4, both the SBS and the ISBS are nonempty in all superadditive QTU games. An interesting question is whether, in such games, these sets contain only, or at least some, Pareto optimal payoffs.

Dutta et al. showed that it is possible that $ISBS \cap v^*(N) = \emptyset$. Specifically, they consider the 4 player TU game (N, μ) , where $N = \{1, 2, 3, 4\}$, $\mu(1, 2, 3) = \mu(2, 3, 4) = 66$, $\mu(1, 4) = 46$, $\mu(1, 2, 4) = \mu(1, 3, 4) = 63$, $\mu(N) = 80$, and $\mu(S) = 0$ otherwise. They show (Dutta et al. 1989, Proposition 4.1) that $CB(N, \mu) = \emptyset$.

In contrast, the SBS in this game does contain Pareto optimal payoffs. For example, $x^* = (23, 17, 17, 23) \in SBS$. Indeed, $\{1, 4\}$, $\{1, 3, 4\}$, and $\{1, 2, 4\}$ cannot block x^* . It remains to check for coalitions $\{1, 2, 3\}$ and $\{2, 3, 4\}$. Now, if $\{1, 2, 3\}$ blocks x^* with y , then $y_1 > x_1^*$, implying that $[y_2 + y_3] < [66 - 23]$. But then, there exists a payoff z such that $z(2, 3, 4) = \mu(2, 3, 4) = 66$ and $y < z$. Since $z(S) \geq \mu(S)$ for all S , $z \in A$. The stability of A implies, therefore, that $y \notin A$. An analogous argument shows that $\{2, 3, 4\}$ cannot block x^* using a payoff in A . Thus, $x^* \in A$.

It remains an open question whether in superadditive TU games we have that $SBS \subset v^*(N)$, or, at least, $SBS \cap v^*(N) \neq \emptyset$. An affirmative answer to this question will be particularly pleasing, since if the "real world" is to provide a guideline, then the notion of the SBS seems to be more appropriate than the ISBS (or the CB); individuals often insist on some compensation if the "status quo" is to be changed.

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